The Study of Turbulent State in Quantum Fluid

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I study the dynamics and statistics of turbulence in quantum fluid such as superfluid $^4$He in which all rotational fluid is carried by quantized vortices.

By numerically solving the nonlinear Schrödinger equation, I obtain the dynamics of quantized vortices in turbulence as tangled state and investigate the statistics like energy spectrum, fractal structure, etc.
Quantized Vortices and Quantum Turbulence

Ordinary fluid (air, water)
- There is viscosity
- From large to small vortices exist in turbulence

Classical fluid (He-II)
- There is no viscosity
- Circulations of vortices are quantized and turbulence is realized as vortex tangle

$\kappa = \frac{m}{\hbar}$
Quantum Turbulence As an Ideal System of Turbulence

Vortices in ordinary turbulence (Navier-Stokes simulation by S. Kida)

- All circulations around vortices have arbitrary value and vortices are indefinite

Vortices in quantum fluid turbulence

- All circulations are quantized and vortices are definite
  \[\rightarrow\] Vortex skeletons in turbulence!
Experiment of Superfluid $^4$He-II


Two-counter rotating disks


Visualization of quantized vortices in turbulence

Energy spectrum of superfluid turbulence

Quantum turbulence obeys the ordinary Kolmogorov law
Proposed Statistics of Vortices and the Energy Spectrum

3 regions: classical, quantum, and dissipative with elementary excitations

Structure of quantized vortices and the energy spectrum are closely related with each other.
Model of Quantum Fluid and Turbulence

Nonlinear Schrödinger equation

\[
[i - \gamma(k)] \frac{\partial \Phi(k)}{\partial t} = \left[ (k^2 - \mu)\Phi(k) + \frac{g}{L^3} \sum_{k_1} \tilde{V}(k_1)\Phi(k - k_1) \right. \\
\left. + \frac{g}{L^6} \sum_{k_1, k_2} \Phi(k_1)\Phi^*(k_2)\Phi(k - k_1 + k_2) \right]
\]

\[
\Phi(x) = |\Phi(x)| \exp[i\theta(x)]
\]
\[
\rho(x) = |\Phi(x)|^2 : \text{Density}
\]
\[
u(x) = 2\nabla \theta(x) : \text{Velocity}
\]
\[
\xi = 1/\sqrt{g\rho} : \text{Vortex core size}
\]

Vortex filament model

\[
\frac{\partial x_0(t)}{\partial t} = v_s(x_0)
\]
\[
v_s(x) = v_{\text{ind}}(x) + v_{\text{sa}}(x)
\]
\[
v_{\text{ind}}(x) = \frac{\kappa}{4\pi} \int \frac{[x_0(t) - x] \times dx_0(t)}{|x_0(t) - x|^3}
\]

Quantized vortex
Numerical Simulation of Nonlinear Schrödinger Equation

Details of Simulation

\[ \tilde{\gamma}(k) = \begin{cases} 
0 & (k < 2\pi/\xi) \\
\gamma_0 & (k \geq 2\pi/\xi)
\end{cases} \quad \text{: Dissipation in scales smaller than } \xi 
\]

\[ |\tilde{V}(k)| = \begin{cases} 
V_0 & (k_0 - \Delta k < k < k_0 + \Delta k) \\
0 & (\text{otherwise})
\end{cases} \quad \text{: Energy injection} 
\]

\[ \xi = 1 : \text{Healing length} \quad g = 1 : \text{Coupling constant} \]
\[ \gamma_0 = 1 \quad V_0 = 50 \quad \Delta k = L/2\pi \quad k_0 = 2\Delta k \]

Space: Fully dealiased pseudospectral-Galerkin method

Time: Runge-Kutta-Gill method
Turbulent State in the Simulation

Periodic box with $256^3$ grids

(stereogram)
Energy Spectrum of Turbulence

Energy spectrum

$k < \frac{2\pi}{l}$: Quantum fluid turbulence shows the Kolmogorov law: there is a similarity between quantum and ordinary fluid.

$k > \frac{2\pi}{l}$: There is Kelvin-wave turbulence characteristic in quantum fluid.
Huge Scale Simulations

In Japan Atomic Energy Agency

Some bottleneck effect between Richardson (Kolmogorov) and Kelvin-wave cascade?
Connection between Richardson and Kelvin-wave Cascade

Two analytical proposals

V. S. L’vov et. al, PRB 76, 024520 (2007).

E. Kozik and B. Svistunov, cond-mat/0703047

Bottleneck region as statistical equipartition.

Complicated vortex bundle structure.

One of the big mystery in quantum fluid turbulence
Summary & Outlook of Quantum Fluid Turbulence

Quantum fluid turbulence consists of quantized vortices and shows the Kolmogorov law.

Quantum fluid turbulence can become an ideal prototype to study turbulence from the view of elementary structure of vortices and the relation between dynamics of vortices and statistics like the Kolmogorov law.
Future Subject

- Details in the region of Kelvin-wave turbulence.
- Calculation of statistical and dynamical properties of vortices in real space, such as size-distribution of vortex loops, fractal dimension of vortex lines, vortex linking number etc.
- Investigation of relation between statistics and dynamics in real space and wave-number space.

MK and M. Tsubota, JPSJ 74, 3248 (2005).