Realization of Quantum Turbulence in Atomic Bose-Einstein Condensation

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Quantum Fluid and Quantum Turbulence

System of quantum fluid and quantum turbulence

- Superfluid $^4\text{He}$ and $^3\text{He}$
- Magnetically or optically trapped ultra-cold Atoms

→ At low temperatures, these systems show inviscid superfluid with Bose-Einstein condensation (or BCS) transition
Quantized Vortex

In quantum fluid, all vortices are quantized with quantum circulation $\kappa = \hbar / m$

- All vortices have same circulation $\kappa = \oint \mathbf{v}_s \cdot d\mathbf{s} = \hbar / m$ around vortex cores.
- Vortex core is very thin ($\sim \text{Å} : ^4\text{He}, \sim 10\text{nm} : ^3\text{He}, \sim 100\text{nm} \text{BEC of cold atoms}) :$ Vortex filament model becomes realistic
Quantum Turbulence From Quantized Vortices

Quantum turbulence can be realized as tangled quantized vortices

Simulation of quantum turbulence by vortex filament model

\[ \frac{\partial \mathbf{x}_0(t)}{\partial t} = \mathbf{v}_s(\mathbf{x}_0) \]

\[ \mathbf{v}_s(\mathbf{x}) = \mathbf{v}_{\text{ind}}(\mathbf{x}) + \mathbf{v}_{\text{sa}}(\mathbf{x}) \]

\[ \mathbf{v}_{\text{ind}}(\mathbf{x}) = \frac{\kappa}{4\pi} \int \frac{[\mathbf{x}_0(t) - \mathbf{x}] \times d\mathbf{x}_0(t)}{|\mathbf{x}_0(t) - \mathbf{x}|^3} \]

Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

\[ \hbar[i - \gamma(x)] \frac{\partial}{\partial t} \Phi(x) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - \mu + \frac{4\pi \hbar^2 a}{m} |\Phi(x)|^2 \right] \Phi(x) \]

\( a \) : Scattering length

\( \gamma(x) \) : Dissipation term for elementary excitations

Equation for dynamics of order parameter in BEC
Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

\[ \hbar[i - \gamma(x)] \frac{\partial}{\partial t} \Phi(x) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - \mu + \frac{4\pi \hbar^2 a}{m} |\Phi(x)|^2 \right] \Phi(x) \]

\[ \Phi(x) = |\Phi(x)| \exp[i\theta(x)] \]

\[ \rho(x) = |\Phi(x)|^2 : \text{Density} \]

\[ \mathbf{v}(x) = (\hbar/m) \nabla \theta(x) : \text{Velocity field} \]

\[ \xi = 1/\sqrt{8\pi a\bar{\rho}} : \text{Vortex core size} \]

Vortex
Quantum turbulence can be realized as tangled quantized vortices

Simulation of quantum turbulence by Gross-Pitaevskii equation
Energy Spectrum of the Gross-Pitaevskii Turbulence

We observed the Kolmogorov law: $E(k) \propto k^{-5/3}$ between scale of injected vortex ring $R$ and the vortex core size $\xi$. 

![Energy spectrum graph](image)
Quantum turbulence can be realized as tangled quantized vortices.


There are some similarities between classical and quantum turbulence.
Kolmogorov Law for Fully Developed Steady Turbulence

Keeping the self-similarity, Energy is transferred from large to small scales without dissipation → Kolmogorov law

\[ E(k) = C \varepsilon^{2/3} k^{-5/3} \]

\( C \): Kolmogorov constant
Richardson Cascade of Vortices

Energy-containing range:
Large eddies are nucleated

Inertial range:
Eddies are broken up to small ones

Energy-dissipative range:
Small eddies are dissipated
Richardson Cascade of Vortices

"big whirls have little whirls,
which feed on their velocity,
and little whirls have lesser whirls
and so on to viscosity—"
Leonardo da Vinci Already Had Same Image

Sketch of eddies in turbulence made by water pipe

Leonardo da Vinci

- Turbulence is constituted by eddies.
- Turbulence classify eddies into size.
- Eddies with same class interact each other.
Eddies in Classical Turbulence

It is very difficult to identify eddies and the Richardson cascade (Eddies are diffused by the viscosity)
Identification of Vortices

Classical turbulence: difficult

Quantum turbulence: already defined as topological defects

Cascade of quantized vortices can be expected in quantum turbulence.

Not only Richardson cascade, but also Kelvin wave cascade is also expected in quantum turbulence.

Vortex dissipates to elementary excitations (This effect is not included in Gross-Pitaevskii equation)
Energy Spectrum of the Gross-Pitaevskii Turbulence

$R$ : Size of injected vortex rings

$E(k) \propto k^{-5/3}$ : Kolmogorov law

$l = (V/L)^{1/2}$ : Vortex mean distance

$E(k) \propto k^{-6}$ : Different scaling from the Kolmogorov law (Kelvin wave turbulence : intrinsic phenomenon of quantum turbulence?)

$\xi$ : Vortex core size
The Study of Quantum Turbulence in the Viewpoint of Quantized Vortices

Quantized vortices give the real Richardson cascade in turbulence.

Cascade of 1 vortex ring in turbulence

What is the relation between cascades in wave number space and real space?

Enstrophy and its spectrum:

\[ Q = \int \, d\mathbf{x} \, |\nabla \times v(\mathbf{x})|^2 = \int \, dk \, k^2 E(k) = \int \, dk \, Q(k) \]
Relation Between Wave Number Space and Real Space

In quantum turbulence, enstrophy is directly related to vortex line length:

\[ Q = \int d\mathbf{x} |\nabla \times \mathbf{v}(\mathbf{x})|^2 = \int dk \, Q(k) \propto \kappa L \quad \text{(Quantum turbulence)} \]

Vortex line length spectrum:

\[ E(k) \propto k^{-5/3} \rightarrow Q(k) \propto L(k) \propto k^{1/3} \]

1. Vortex length by the size of vortex ring

2. Fractal length
The Study of Quantum Turbulence by Superfluid Helium

Quantum turbulence has been realized only in the system of superfluid helium.

Vibrating wire


Two-counter rotating disks (Paris)

Oscillating grid (Lancaster)

D. L. Bradley et al. Phys. Rev. Lett. 96, 035301 (2-6)
Observation of Quantized Vortices

• (Second) sound
• Vibrating wire
• NMR second peak

Only total vortex line length can be measured


Visualization of vortex lattice under the rotation

It is very difficult to measure the spatial distribution of quantized vortices
Atomic Bose-Einstein Condensates and Quantized Vortices

- Trapped atomic gas
- Laser cooling
- BEC

Evaporation cooling
Observation of Vortex Lattice Under the Rotation

Rotation of BEC

Rotation of anisotropic potential


Optical spoon
Observation of Vortex Lattice Under the Rotation

P. Engels, et al.  
PRL 87, 210403 (2001)

J.R. Abo-Shaeer, et al.  
Science 292, 476 (2001)

V. Bretin et al.  
PRL 90, 100403(2003)

K. W. Madison et al.  
PRL 86, 4443(2001)

M. R. Matthews et al.  
PRL 83, 2498(1999)
The Study of Quantum Turbulence in Atomic BEC

There has been no research of quantum turbulence in this field.

The merit of Atomic BEC
- Almost all physical parameters can be controllable such as the total number of particles, the temperature, the density, and even inter-particle interaction.
- Quantized vortices can be observed as holes of the density.

Atomic BEC can be a good candidate to study quantum turbulence (Human being can get controllable turbulence!)
Toward the Realization of Quantum Turbulence

It is difficult to apply the velocity field to atomic BEC → Effective tool: precession rotation

- Single rotation along one axis is realized without rotation along the other axis.
- Rotating vortex lattice can be realized when second rotation is weak.
- Rotating lattice becomes unstable and enters turbulence when second rotation is strong.

Precession Rotation in Atomic BEC

It is no need to rotate the experimental system itself for the case of atomic BEC.

Precession rotation of optical spoon

It is even possible to realize three axes rotation (more isotropic)
Numerical Simulation of the Gross-Pitaevskii Equation

\[ \hbar [i - \gamma(x)] \frac{\partial}{\partial t} \Phi(x) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - \mu + U(x) + \frac{4\pi\hbar^2 a}{m} |\Phi(x)|^2 - \Omega(t) \cdot L(x) \right] \Phi(x) \]

\( U(x) \): Magnetic trapping potential
\( \Omega(t) \): Angular velocity of rotation
\( L(x) \): Angular momentum operator
\( \gamma(x) \): Dissipation term for elementary excitations

Equation for dynamics of order parameter in BEC
Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

\[
\hbar [i - \gamma(x)] \frac{\partial}{\partial t} \Phi(x) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - \mu + U(x) + \frac{4\pi \hbar^2 a}{m} |\Phi(x)|^2 - \mathbf{\Omega}(t) \cdot \mathbf{L}(x) \right] \Phi(x)
\]

Precession rotation

\[
\Omega(t) = (\Omega_x, \Omega_z \sin \Omega_x t, \Omega_z \cos \Omega_x t)
\]
Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

\[ \hbar[i - \gamma(x)] \frac{\partial}{\partial t} \Phi(x) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - \mu + U(x) + \frac{4\pi \hbar^2 a}{m} |\Phi(x)|^2 - \Omega(t) \cdot L(x) \right] \Phi(x) \]

Anisotropic trapping potential

\[ U(x) = \frac{m\omega^2}{2} [(1 - \epsilon_1)(1 - \epsilon_2)x^2 + (1 + \epsilon_1)(1 - \epsilon_2)y^2 + (1 + \epsilon_2)z^2] \]
Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

\[ \hbar [i \gamma(x)] \frac{\partial}{\partial t} \Phi(x) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - \mu + U(x) + \frac{4\pi \hbar^2 a}{m} |\Phi(x)|^2 - \Omega(t) \cdot L(x) \right] \Phi(x) \]

Dissipation by the elementary excitation

\[ \gamma(k) = \gamma_0 \theta(k - 2\pi/\xi_0) \]

: Effective in the scales smaller than the vortex core

MK & MT, PRL. 97, 145301 (2006)
Vortex Lattice Simulation

$^{87}\text{Rb}$ atoms: $m = 1.46 \times 10^{-25}$ kg, $a = 5.61$ nm

$N = 2.50 \times 10^5$, $\epsilon_1 = 0.05$

$\omega_x = \omega_y = 120 \times 2\pi$ Hz, $\omega_z = 20 \times 2\pi$ Hz

$\Omega = 0.75 \omega_x$

Quantum Turbulence Simulation

$^{87}$Rb atoms: $m = 1.46 \times 10^{-25}$ kg, $a = 5.61$ nm

$N = 2.50 \times 10^5$, $\omega = 150 \times 2\pi$ Hz

$\Omega_z = \Omega_x = 0.6\omega$, $\epsilon_1 = \epsilon_2 = 0.025$

Numerics: Space: Grid $512^3$ with Dirichlet boundary

$(\text{Chebyshev+tau})^3$, $V = 14.0^3$ $\mu$m Volume

Time: 4th ordered Runge-Kutta

Initial condition: No rotation and anisotropy
Vortices are not crystallized but tangled.
Quantum Turbulence Simulation

Energy spectrum

\[ E_{\text{kin}}^i(k) = \frac{\hbar \omega}{m} \]

\[ C \varepsilon^{2/3} k^{-5/3} \]

20 ensemble average

\[ 2\pi \alpha_{\text{trap}} / R_{\text{TF}} \]

\[ 2\pi \alpha_{\text{trap}} / \xi \]
Quantum Turbulence Simulation

Starting from vortex lattice
Three Axes Rotation

Vortex tangle becomes more isotropic
Three Axes Rotation

Agreement with Kolmogorov law becomes better

\[ E_{\text{kin}}^i(k) m / \hbar \omega a_{\text{trap}} \propto k^{-\eta} \]

\[ \eta = 1.78 \pm 0.194 \]

\[ E_{\text{kin}}(k) m / \hbar \omega a_{\text{trap}} \propto k^{-\eta} \]

\[ \eta = 1.69 \pm 0.037 \]
Summary

• Quantum turbulence is the good system to study turbulence because quantized vortices can be clearly identified (studying the Richardson cascade, the relation between cascade in wave number space and real space).

• Atomic Bose-Einstein condensation is the good experimental system to study quantum turbulence.
Experimental Observation of the Kolmogorov Law

Expansion of BEC after switching off the magnetic trapping

\[ E \propto \int dk \, k^{-5/3} \]

\[ N \propto \int dk \, k^{-11/3} \rightarrow N(k) \propto k^{-11/3} \]

\[ v \sim k \rightarrow N(v) \propto v^{-11/6} \]

\[ v \sim r(\text{TOF}) \rightarrow N(r) \propto r^{-11/6} \]

Density distribution

Two-dimensional projection of vortex configuration

\[ L(k) \propto k^{1/3} \]

\[ \rightarrow \int dz \, L \cos \phi \propto \int dk \, k^{-5/3} \]
Bragg Spectroscopy

Bragg spectroscopy with focused laser beam

$\omega_1, k_1$  $\delta \omega, \delta k$  $\omega_2, k_2$

Collective excitation of BEC

Spatial distribution of velocity field